CERC 2018

Presentation of solutions

December 2, 2018

Silence of the Lamps

- Note that there are only 16850052 ($\approx$ 1.7e7) such triples for 10^6
- You can safely iterate over all solutions, so just use three cycles to find out number of occurences of every possible volume.
- Now do a prefix sum and you're good to go.
- This way you can precompute all results at once and then simply retrieve a result for every query in $O(1)$.

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Complexity is "no worse" than $O(b^{\log_2 b})$.

Matrice

- The biggest size of one kind of triangle on coordinates **x**, **y** could be found as 1 more than biggest triangle on coordinates [x-1][y] and [x][y-1].
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Complexity $O(N^2)$

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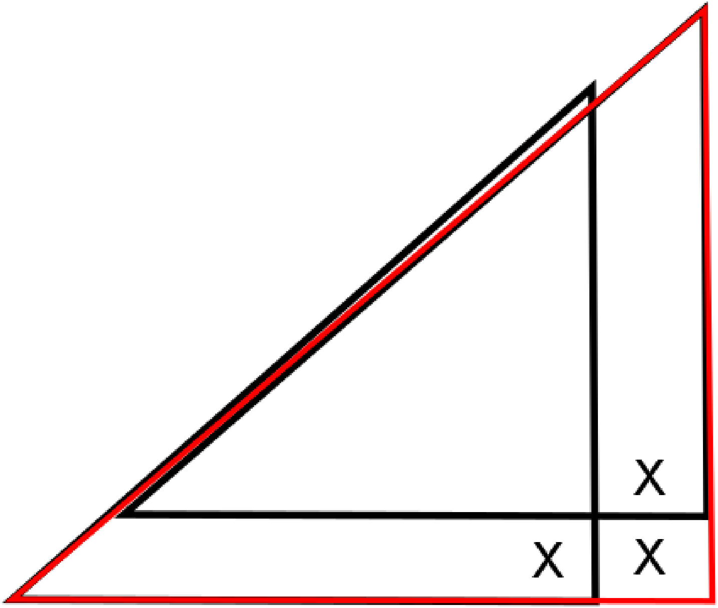

Tree Gump

- Pick the lower leftmost point (or some other extreme).
- Recursively, sort the points by angle and divide them to children according to their sizes.
- Note that if this is done properly, the "root" of tree is always some extreme and the rest is in some kind of "fan" (the points are on one side of it).

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Complexity $O(N^2 \log N)$

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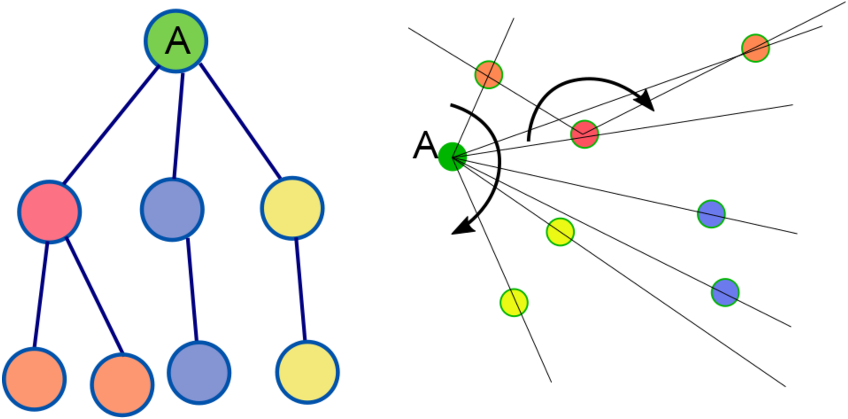

Stones

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- Firstly let us take look on how to solve this problem if $A == B$
- That would be an impartial game solvable with nimbers.
- Actually it is not hard to observe that the behaviour of such nimber sequence – it repeats in modulo A (and B obviously).
- But what if $A \neq B$? The game is impartial!
- Note that if the size of the biggest pile is less or equal than minimum of A and B, it is still an impartial game.

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- An obvious observation is that player with A has an advantage.
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- An obvious observation is that player with A has an advantage.
- In fact he could use principle of tied hand, thinking $A == B$ (in this case we could use nimbers again)
- Lets say player with A is playing:
- Imagine the sum of games is not zero. In this case he could simply follow the rules of nim with $A == B$ and win.
- If the sum of games is zero, he can play a 0-move (not changing the xor) and continue playing as if $A == B$ after that.

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- But what if B is plaing first?
- Note that if there are at least two piles bigger than B, then fate of the second player is sealed since he can't escape player A playing the 0-move.
- But as soon there is just one such pile, he does have a chance – but he certainly has to make a move to that particular pile.
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Complexity $O(N)$

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- Each of them finds the longest string from dictionary which starts on given index and the rest is common.
- Once you figure out how to find the longest string, what remains is to greedily iterate and find the length of the longest string which can be added to already matched part.

The ABCD Murderer - Jury solution

- Create Aho–Corasick automaton using all strings from dictionary.
- We don't need the exact indices of matches, we care only about the length of longest word which can be matched and ends at given index of the ransom text. That can be precomputed in linear time.
- Overall complexity is linear to sum of lengths of the input strings.

The ABCD Murderer - Example

- Ransom note: abecedadabra
- Dictionary: abec, ab, ceda, dad, ra

- |abecedadabra
- abec|edadabra
- abeceda|dabra
- abecedad|abra
- abecedadab|ra
- abecedadabra|

The ABCD Murderer - Alternative approach

- Create suffix array from all concatenated strings from dictionary.
- Make LCP array.
- Iterate over it, keeping minimum from last dictionary string to find the number.

Complexity depends on SA – best $O(N)$, but $O(N \log^2 N)$ is also sufficient

The ABCD Murderer - Alternative approach 2

- Divide strings on input by those which are longer then $\sqrt{N}$ and the rest.
- For those which are longer use KMP.
- The shorter shall be put into a trie.
- Afterward test each index of text in trie.

Complexity of this approach is $O(N\sqrt{N})$, also ok

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- Observe that there is no more than $\sqrt{N}$ distinct pattern lengths.
- We also recommend double hashing!

Complexity of this approach is $O(N\sqrt{N})$

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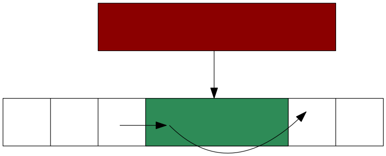
Complexity $O(N \cdot M \cdot \alpha(N) + Q \cdot N \cdot \alpha(N))$



Dog

$$s(t) = s + vt + at^2/2$$

Compute the time it takes the dog to reach the top of his jump (set $s = 0, v = 0$).

$$s(t) = at^2/2$$

$$t_{topdog} = \frac{2 \cdot s(t)}{a}$$

Dog - by binary search

- Observe that the dog will always jump with maximal velocity.
- Binary search for the earliest time when the dog can get the frisbee – we have exact position of the frisbee:

$$X = V_f \cdot \min(T - T_f, F(H_f))$$

$$Y = H_f - (T - T_f)^2/2$$

- X coordinate ok iff $X/V_d \leq T - T_d$.
- Y coordinate ok iff $Y/V_d \leq T - T_d$.
- Check if dog can get to the frisbee in time – check X and Y coordinates separately.

Complexity $O(1)$

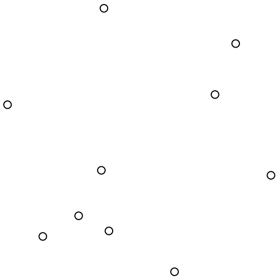
Dog - by exact formula

- There are 3 main cases:
  - frisbee will fall to the ground and dog will fetch it,
  - dog will have to wait until he can jump for frisbee,
  - dog can jump for frisbee immediately.
- Compute minimal time to cover X and Y distance separately and take minimum.

Complexity $O(\log U)$ where $U = 10^{15}$.

The Incredible Hull

- Observe the behavior of the process:



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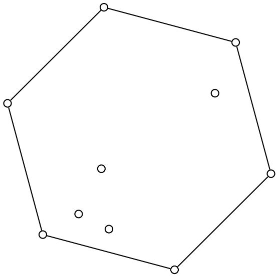

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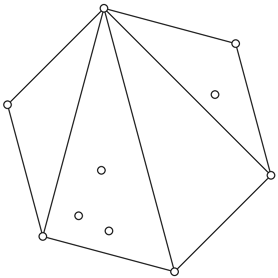

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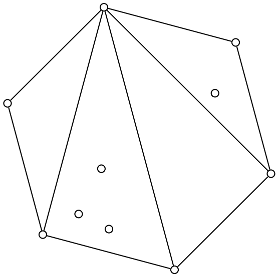

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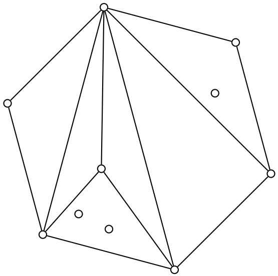

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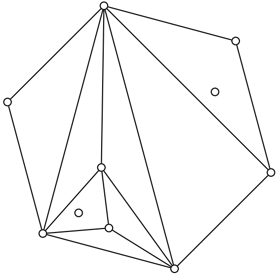

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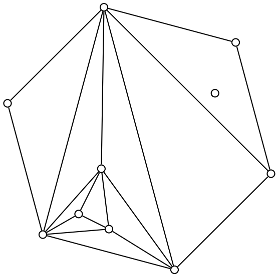

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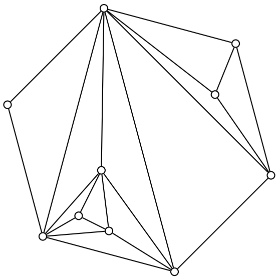

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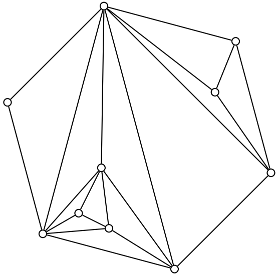

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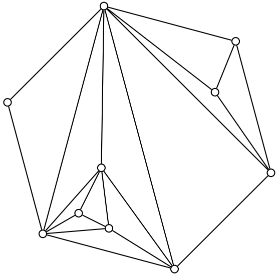

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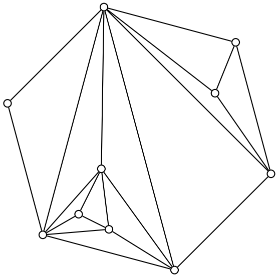

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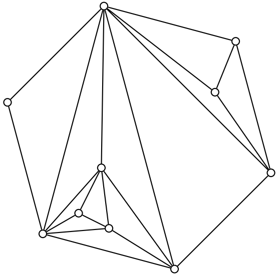

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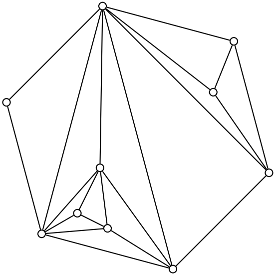
Complexity $O(N \log N)$



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- For each edge, find the time it started to exist and ceased to exist (or possibly infinite).
- Now - let us proceed Divide et Impera over queries, pointing into the middle and dividing queries into those with lesser time and those with bigger time.
- First step is to recursively solve the left half.
- In second step, tak all edges which are in the left half, and either use DSU or ignore them (depending on time of end).
- As you can observe, the queries in half from time t_b to t_e operate with at most $(t_e - t_b + 1) \cdot 2$ nodes. This means that if we would do the DSU "sparse" we would be able to copy it in each node with overhead linear with size of timelapse.
- If you have only one query and it is question, then simply check whether they are both in same component.

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- Homework: Can we solve it online? ;-)

Lord of the Kings

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- Finding minimal number of tiles to host a helipad is equal to finding a Steiner tree on such graph with cities and palace as terminals.
- This can be solved by parametrized DP.

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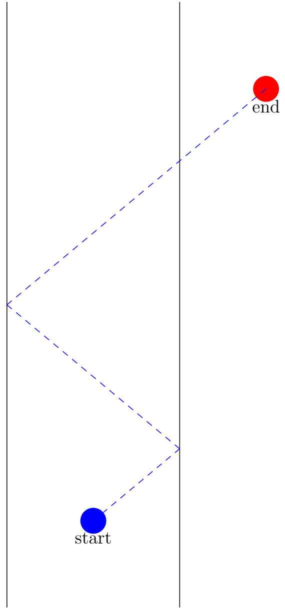
This leads to complexity of $O(N \cdot 3^T + N^2 \cdot 2^T + N^3)$

Mirrority report



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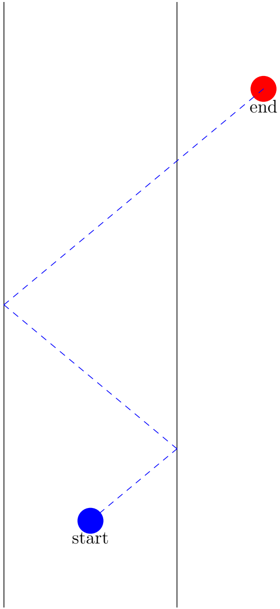

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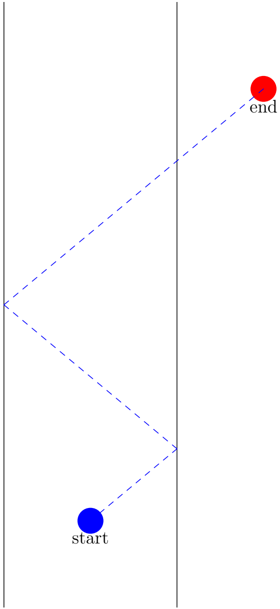

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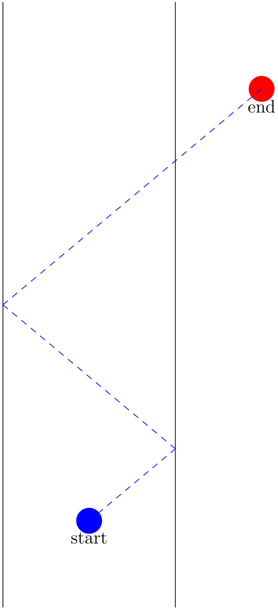

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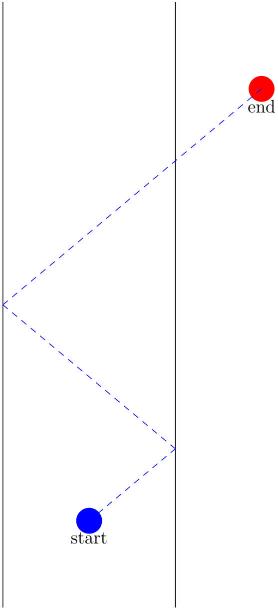

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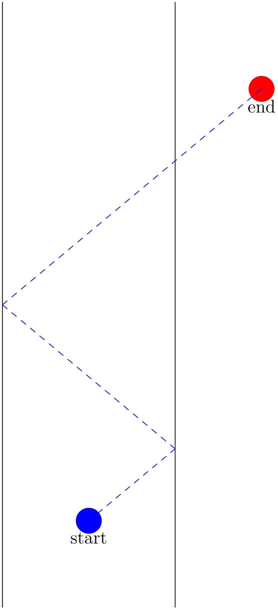

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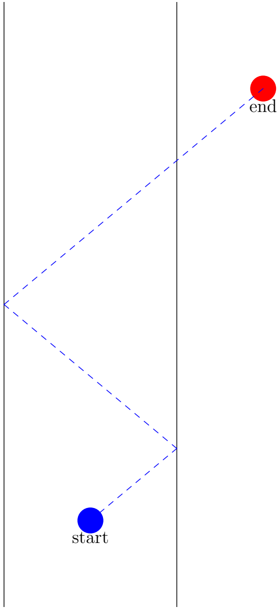

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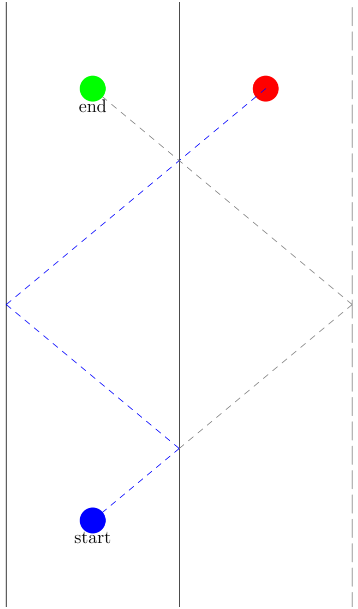
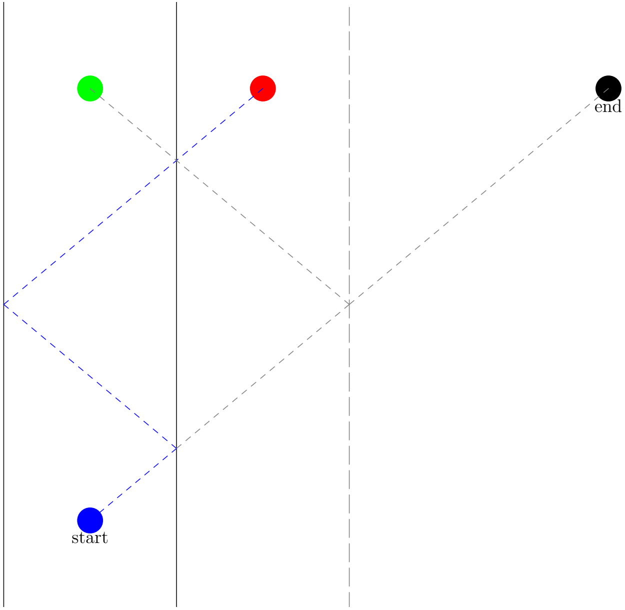
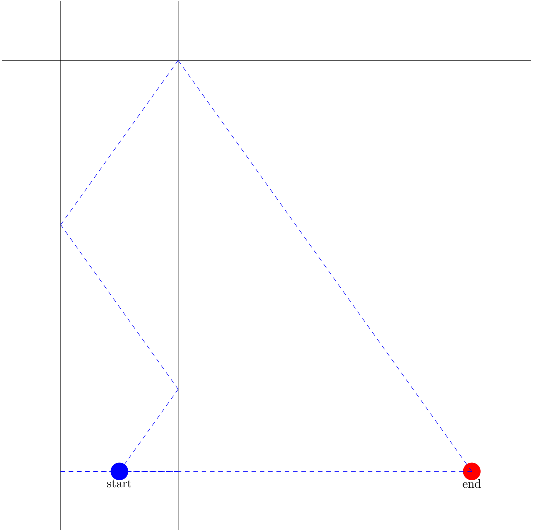


Mirrority report



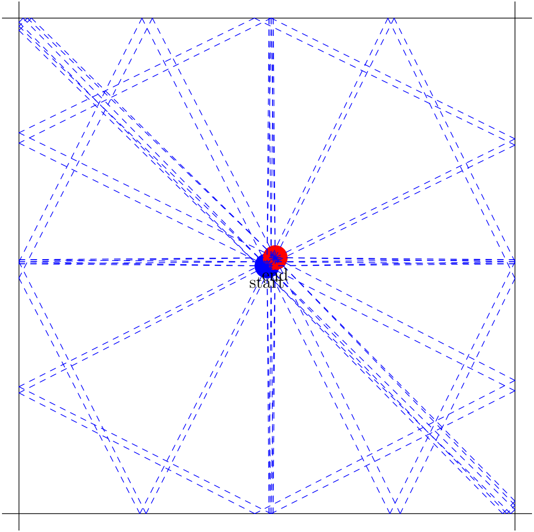
Mirrority report

Special cases – corner which is already gone

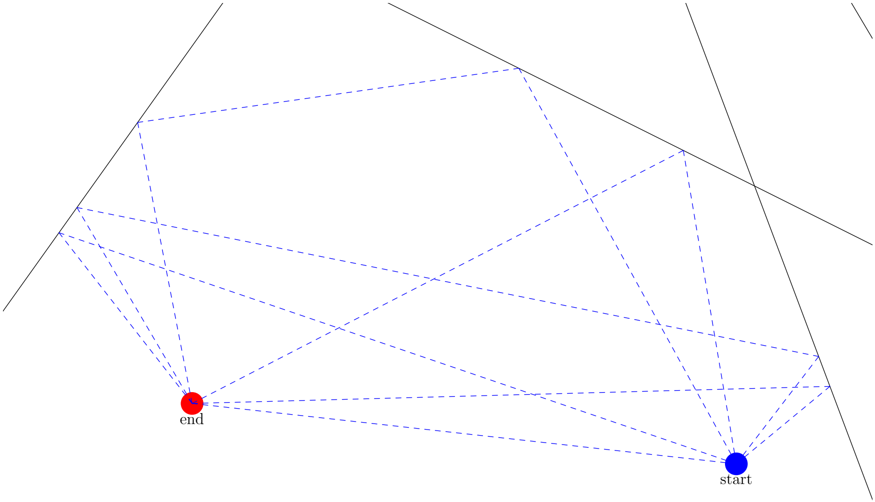


Mirrority report

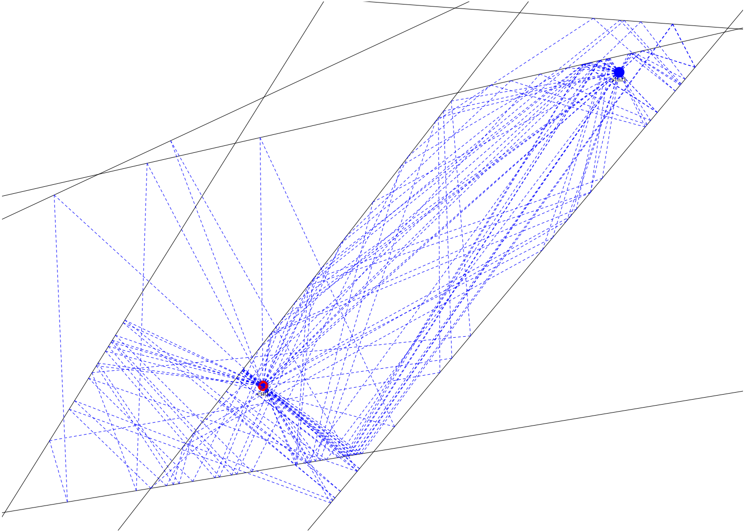
Special cases – ignore corners correctly



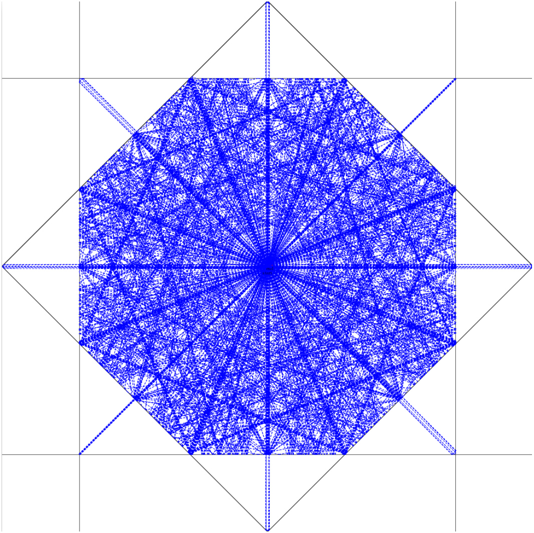
Mirrority report – some cases for amusement



Mirrority report – some cases for amusement



Mirrority report – some cases for amusement



Thank you for your attention!